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Sheath effects on the radiation from biconical antenna immersed in a compressible plasma*

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This paper deals with the studies of sheath effects on the radiation from biconical antenna immersed in a compressible plasma Necessary equations have been developed and calculations have been made for the radiation resistance as a function of sheath thickness

1 Introduction

It is now well established that when an antenna is immersed in a compressible plasma, in addition to e m wave the electroacoustic waves are also being radiated Considerable investigation on such a problem has already been done by Wait & Seshadri The paper written by these authors are too numerous to quote here. However, recently in the study of probes immersed in plasma, the ion sheath surrounding the antenna are also found to affect much the radiation characteristics of antennas (Cohen 1961) In this paper we have analyzed a biconical antenna immersed in a compressible plasma and in this analysis the effects of sheath are duly taken into account.

2 Theory

Taking the assumption that there is no magnetic field, no collision, high frequency and that linearized fluid model can be applied to present problem, we write the following fundamental equations as

$$\nabla \times \boldsymbol{E} = -j\omega\mu_0 \boldsymbol{H} \quad \ldots (1)$$

$$\nabla \times \boldsymbol{H} = j\omega\xi_0 \boldsymbol{E} + n_0 e \boldsymbol{V} \quad \ldots (2)$$

$$j\omega m n_0 \boldsymbol{V} = n_0 e \boldsymbol{E} - \nabla p \quad \ldots (3)$$

$$u^2 m n_0 \nabla \boldsymbol{V} = -j\omega p, \quad \ldots (4)$$

where $\boldsymbol{V}$ is the deviation from the average of electron velocity p is the pressure m is the mass of electron, e is the charge on electron and u is the root mean square of the thermal velocity of electron

* Part of the research work was done by one of the authors (D.C A) when he was at the Department of Electrical Engineering, Tohoku University, Sendai, Japan during 1973-74.

** Present address ; Department of Physics, J. Christian College, Allahabad, Taking the spherical coordinates, eq (4) can be represented as

$$\frac{u^2 mn}{\gamma^2 \sin\theta}\left\{\sin\theta\frac{\partial}{\partial\gamma}(r^2 V_\gamma)+\gamma\frac{\partial}{\partial\theta}(V_\theta \sin\theta)\right\} = -j\omega p \tag{5}$$

where it is assumed that $V_\theta = 0$

Now substituting the value of V from eqs (2) and (3) into eq (5) we get an equation for pressure given by

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial p}{\partial r}\right)+\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial p}{\partial\theta}\right)+k_P^2 r^2 p = 0 \qquad \ldots \quad (6)$$

Since there is no variation of p in the ϕ direction we can have the condition that

$$p(r,\theta) = R(r).\Theta(\theta), \qquad \ldots \quad (7)$$

which being substituted into eq (6) gives

$$\frac{d}{dr}\left(r^2\frac{dR}{dr}\right)+k_P^2 r^2 R-n(n+1)R = 0, \qquad \ldots \quad (8)$$

$$\frac{d}{d\theta}\left(\sin\theta\frac{d\Theta}{d\theta}\right)+n(n+1)\sin\theta\,\Theta = 0, \qquad \ldots \quad (9)$$

$$\left.\begin{aligned} R(r) &= P\frac{e^{-jk_P r}}{r}+Q\frac{e^{-jk_P r}}{r}, && \text{for } n = 0 \\ &= A_n j_n(k_P r)+A'_n n_n(k_P r), && \text{for } n \neq 0 \end{aligned}\right\} \qquad \ldots \quad (10)$$

$$\left.\begin{aligned} \Theta(\theta) &= \log\tan\frac{\theta}{2}, \quad n = 0 \\ &= C_n P_n(\cos^2\theta')+D_n Q_n(\cos\theta), \; n \neq 0, \; n = \text{integer} \\ &= C_n P_n(\cos\theta)+D_n P_n(-\cos\theta), \; n \neq 0, \; n \neq \text{integer} \end{aligned}\right\} \qquad \ldots \quad (11)$$

Here $J_m(z)$, $n_n(z)$ are the spherical Bessel function, $P_n(x)$, $Q_n(x)$ are the Legendre function

Now separating out the electron plasma mode from eqs (2) and (3) and using the relation (7), following expressions for the electric field of electron plamsa wave is derived

$$E_r^p = -\frac{e}{m\epsilon\omega^2}\frac{dR(r)}{dr}\Theta(\theta), \qquad \ldots \quad (12)$$

$$E_\theta^p = -\frac{e}{m\epsilon\omega^2 r}R(r)\frac{d\Theta}{d\theta} \qquad \ldots \quad (13)$$

By selecting the proper solution from eqs. (10) and (11) and substituting into eqs. (12) and (13) the equation of the electric field for electron plasma wave is derived. Now we study the electric field for the case shown in figure 1 for the regions I and II which are separated from each other by the spherical surface $r = l$ as shown in the figure.

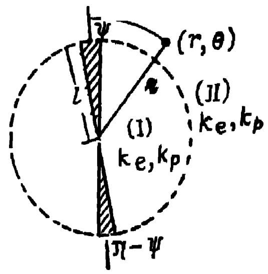

Fig 1 Representation of inner and outer regions I and II respectively.

Case 1 : Electric Field of E.P. Wave in the Region I :

In the region I which includes the antenna, $E_r^p = 0$ at $\theta = \psi, \pi - \psi$ and so from eq (12) we get the following expressions i e ,

$$\Theta(\psi) = 0 \quad \text{and} \quad \Theta(\pi - \psi) = 0 \qquad \ldots \quad (14)$$

In order that the constants of eq (11) may satisfy the eq (14) we get the condition that $D_n = \pm C_n$ Now using the negative sign that is the biconical antenna being fed in balance eq. (15) may be obtained as

$$\Theta(\theta) = \tfrac{1}{2}\{P_n(\cos\theta) - P_n) - \cos\theta)\} = M_n(\cos\theta) \qquad \ldots \quad (15)$$

Now, for $R(r)$, $A'_n = 0$ must be satisfied because when $r \to 0$, $n_n(k_p r)$ in eq (10) is infinity in order of $r^{(r+1)}$ and so we get the following equation

$$R(r) = A_n J_n(k_p r) \qquad \ldots \quad (16)$$

and thus the electric field for electron plasma wave in region I can be represented as

$$E_r^p = \frac{-ek_p}{m\epsilon\omega^2} \sum_n A_n \frac{J'_n(k r)}{J_n(k_p l)} M_n(\cos\theta) \qquad . \quad (17)$$

$$E_\theta^p = -\frac{e}{m\epsilon\omega^2} \frac{1}{r} \sum_n \frac{A_n J_n(k_p r)}{J_n(k_p l)} \cdot \frac{d}{d\theta} Mn(\cos\theta) \qquad .. \quad (18)$$

Here we can consider that the mode of $n = 0$ does not exist practically because at $n = 0$, $(rE\ )_{r \to 0} = \infty$

Case 2 : Electric Field of E.P. Wave in Region II :

Now in order that the electric field in the interior part of region II is infinite, order of **H** is integer and function of **H** must be first kind Legendre function and have only odd mode by considering the balanced type. Also $R(r)$ is given by second kind spherical Hankel function $h_k(k_p r)$ in order that the electric field as $e^{-jk_p r}/r$ at the enough far region. Consequently from eqs. (12) and (13) the electric field of the second region can be represented as

$$E_r{}^p = \frac{-ek_p}{mc\omega^2} \sum_{1,3,\ldots} \frac{B_K h_k{}^{(2)\prime}(k_p r)}{h_k{}^{(2)}(k_p l)} \cdot P_k(\cos\theta), \qquad \ldots (19)$$

$$E_\theta{}^p = \frac{-e}{m\epsilon\omega^2} \cdot \frac{1}{r} \sum_{1,3\ldots} \frac{B_R h_k{}^{(2)}(k_p r)}{h_k{}^{(2)}(k_p l)} \cdot \frac{d}{d\theta} P_k(\cos\theta) \qquad \ldots (20)$$

where B_K is constant. For example the electric field at a far region enough to be $\gamma \rightarrow \infty$ is approximated by $E_r{}^p$ as the electric fields are reduced in the type of $E_r{}^p \alpha \frac{1}{r}$ and $E_\theta{}^p \alpha \frac{1}{r^2}$ respectively

Case 3 · Electromagnetic Field of E'M Wave in a Plasma

By using the expression of e.m field from a biconical antenna given by Schelkunoff (1952) we got the following expressions for the electromagnetic field and are represented as

$$\text{region I} \begin{cases} r^2 E_r{}^e = -\dfrac{1}{2\pi j\omega c} \sum\limits_r a_n \dfrac{\hat{J}_n(k_e r)}{J_n(k\,l)} \quad M_n(\cos\theta) & \ldots (21) \\[2ex] rE_\theta{}^e = \dfrac{n\ V(r)}{2\pi \sin\theta} + j\dfrac{n_e}{2\pi} \sum\limits_n \dfrac{a_n \hat{J}_n(k_e r)}{n(n+1)\hat{J}_n(k_e l)} \dfrac{d}{d\theta} M_n(\cos\theta) & \ldots (22) \\[2ex] rH_\phi = \dfrac{I_0(r)}{2\pi \sin\theta} + \dfrac{1}{2\pi} \sum\limits_n \dfrac{a_m}{n(n+1)} \dfrac{\hat{J}_n(k_e r)}{\hat{J}_n(k_e l)} \dfrac{d}{d\theta} Mn(\cos\theta) & \ldots (23) \end{cases}$$

$$\text{region II} \begin{cases} r^2 E_r{}^e = -\dfrac{1}{2\pi j\omega c} \sum\limits_{1,3\ldots} b_k \dfrac{\hat{H}_k(k_e r)}{\hat{\mathrm{H}}_k(k_e l)} \ P_k(\cos\theta) & \ldots (24) \\[2ex] rE_\theta{}^p = j\dfrac{n}{2\pi} \sum\limits_{1,3} \dfrac{b_k}{k(k+1)} \cdot \dfrac{\hat{H}'_k(k_e r)}{\hat{H}_k(b_e l)^*} \cdot \dfrac{d}{d\theta} P_k(\cos\theta) & \ldots (25) \\[2ex] rH_\phi = \dfrac{1}{2\pi} \sum\limits_{1,3\ldots} \dfrac{b_k}{k(k+1)} \cdot \dfrac{H_k(k_e r)}{H_k(k_e l)} \dfrac{d}{d\theta} P_k(\cos\theta) & \ldots (26) \end{cases}$$

where

$$n_e = 120\pi \left(1 - \frac{\omega^2 p}{\omega^2}\right)^{-\frac{1}{2}}, \ K_e = \frac{n_e}{\pi} \log\cot\frac{\psi}{2},$$

$$\hat{H}_k(z) = Zh_k{}^{(2)}(z), \ \hat{H}_k(z) = \frac{d}{dz}\hat{H}_k(z).$$

3. Input Impedance of Enough Thin Biconical Antenna

3.1 *General Expression of Input Impedance*

Considering that the electromagnetic field of the centre fed biconical antenna is transmitted till the top of antenna ($r = l$) by TEM mode and it generates higher order mode for both the e.m. and e.p. waves respectively, the input impedance of biconical antenna in a plasma can be analyzed by mode analysis method of impedance given by Schelknuff (1952) The input impedance of the antenna considering it as a transmission of length βl and loaded by impedance Z_t (which includes the impedance for e.p and e.m wave) can be represented by

$$Z_i = \frac{KZ_t \cos\beta l + jK \sin\beta l}{K \cos\beta l + jZ + \sin\beta l}. \qquad (27)$$

where

$$K = \frac{n}{\pi} \log \cot \frac{\psi}{2}.$$

In order to obtain Z_t we integrate eq (23) for θ from $\theta = \psi$ to $\theta = \pi - \psi$ at $r = l$ in the region I and from the condition that the magnetic fields in the second region is equal at $r = l$, the impedance at the top of the antenna can be expressed by

$$\frac{1}{Z_t} = -\frac{n}{\pi K V(l)} \sum_{1,3} \frac{b_k}{k(k+1)} P_k(\cos\psi) \qquad \dots (28)$$

Now, if the inverse of the top impedance is expressed by

$$Z^m = \frac{K^2}{Z_t}, \qquad \dots (29)$$

then from eq (27) input impedance of enough thin antenna i e $K \to \infty$ can be expressed as

$$Z_i \to \frac{Z^m}{\sin^2\beta l} - jK \cot\beta l. \qquad \dots (30)$$

3.2 *All regions of antenna surrounded by sheath.*

First we wish to obtain the input impedance of model where antenna region I is surrounded by spherical plasma sheath and the outer region II is filled to extent by plasma The boundary condition of the tangential component of e m. field at the boundary $r = l$ is given by-

$$l(E_\theta^e)^{(I)} = l(E_\theta^e + E_\theta^{pII}) \qquad \dots (31)$$

The geometry of the situation is shown in figure 2. By substituing the fields from eqs. (19) to (26) for e m wave and e p wave into eq (31) an equation for the constant b_k can be derived. Now, if the antenna is very thin then the angle ψ of the cone tends to zero and so the approximate value of order n becomes

$$n = k + \frac{1}{\log \frac{2}{\psi}} \qquad k = 1, 3 \ldots \tag{32}$$

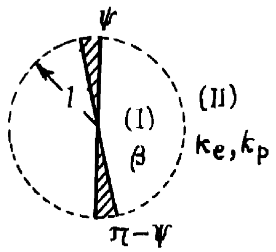

Fig. 2. Inner region I surrounded by sheath and the outer II by the plasma

Then neglecting the higher order terms i.e. $\left[\frac{1}{\left(\log\frac{2}{\psi}\right)^2}\right]$ we can have

$$b_k = -\frac{n\beta v(l)}{K_\beta}(2K+1)|Z_k + \epsilon_k - Z_k^{\beta}|^{-1} \tag{33}$$

$$\epsilon_k = \left(1 - \frac{2\pi e k(k+1)}{Z_k^{+} m\epsilon_0\omega^2}\,\frac{B_k}{b_k}\right) \quad \ldots \tag{34}$$

$$Z_k^{+} = j\eta_e \frac{\hat{H}_k'(k_e l)}{\hat{H}_k(k_e l)} \quad \ldots \tag{35}$$

$$Z_k^{\beta} = j\eta_\beta \frac{\hat{J}_k'(\beta l)}{\hat{J}_k(\beta l)} \quad \ldots \tag{36}$$

In eq. (34) B_k/b_k is the ratio of e.m. wave to e.p. wave in the outer region II and is obtained by using the boundary conditions at $r = l$.

In this time the boundary condition used is given by

$$e\boldsymbol{n}\cdot\boldsymbol{E} = \left(j\omega m + \frac{\alpha}{j\omega}\right)\boldsymbol{n}\cdot\boldsymbol{V} \quad \ldots \tag{37}$$

which includes the $\alpha(\alpha = \infty$, rigid boundary condition) that generally shows the hardness of boundary. Now the relationship between the electron velocity and the electric field may be expessed as

$$\boldsymbol{V}^e = \frac{e}{jm\omega}\boldsymbol{E}^e, \quad \ldots \tag{38}$$

$$\boldsymbol{V}^p = \frac{\epsilon_0\omega}{jen}\boldsymbol{E}^p \quad \ldots \tag{39}$$

so that from the electric field equation for r direction i.e. eq. (19), (24) and the relations (37) to (39) we get the following equations as

$$\frac{B_k}{b_k} = \frac{jen_0}{2\pi \omega \epsilon_0 k_p l^2} \cdot \frac{k_k^{(2)}(k_p l)}{h'_k{}^{(2)}(k_p l)} U, \qquad \ldots \quad (40)$$

$$u = \frac{}{\alpha + m(\omega_p{}^2 - \omega^2)} \qquad \ldots \quad (41)$$

Consequently using the above result the constant in eq. (33) is obtained as

$$b_k = \frac{jV_\beta(l)}{K_\beta} (2k+1)[\delta_k(u)]^{-1}, \qquad \ldots \quad (42)$$

where

$$|\delta_k[u]] = \frac{n_e}{\eta_\beta} \frac{\hat{H}'_k(k_e l)}{\hat{H}_k(k_e l)} - \frac{\hat{J}_k{}'(\beta l)}{\hat{J}\ (\beta l)} - u\left(\frac{w_p}{w}\right)^2 \frac{n_e k(k+1)}{\eta_\beta k_e l k_p l}$$

$$\frac{h_k{}^{(2)}(k_p l)}{h_k{}^{(2)\prime}(k_p} \qquad \ldots \quad (43)$$

Furthermore, the inverse impedance Z^m of top impedance given in eq. (29) is expressed as eq (44) when the antenna is enough thin and the boundary is rigid $(U = 1)$

$$Z^m = -j\frac{n_\beta}{\pi} \sum_{1,3} \frac{2k+1}{k(k+1)} [\delta_k(1)]^{-1}. \qquad \ldots \quad (44)$$

Eq. (44) also expresses the impedance at $\beta l = (\pi/2)$ from the top of antenna.

3.3 *When sheath region includes not only the interior region but also the outer region*

The geometry of the problem is as shown in figure 3. Here we consider the case where the thickness of the sheath or dielectric across the top of the antenna

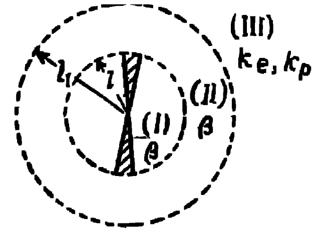

Fig. 3, Regions I and II surrounded by sheath and the region III by the plasma.

includes not only the interior region but also the outer region. In this case the e.m. fields are expressed by

Region II
$$\left.\begin{aligned} 2\pi k(k+1)rE_\theta &= j\eta_\beta \left\{ b_k \frac{\hat{H}_k'(\beta r)}{\hat{H}_k(\beta l)} + C_k \frac{\hat{J}'_k(\beta r)}{\hat{J}_k(\beta l)} \right\} P_k'(\cos\theta) \\ 2\pi k(k+1)rH_\varphi &= \left\{ b_k \frac{\hat{H}_k(\beta r)}{\hat{H}_k(\beta l)} + C_k \frac{\hat{J}_k(\beta r)}{\hat{J}_k(\beta l)} \right\} P_k'(\cos\theta) \end{aligned}\right\} \quad \begin{aligned} &\cdots (45) \\ &\cdots (46) \end{aligned}$$

Region III
$$\left.\begin{aligned} 2\pi k(k+1)rE_\theta &= j\eta_e \frac{\hat{H}_k'(k_e r)}{\hat{H}_k(k_e l_1)} d_k \varepsilon^{(r)} P_k'(\cos\theta) \\ 2\pi k(k+1)\gamma H_\varphi &= d_k \frac{\hat{H}_k(k_e l)}{\hat{H}_k(k_e l)} P_k'(\cos\theta) \end{aligned}\right\} \quad \begin{aligned} &\cdots (47) \\ &\cdots (48) \end{aligned}$$

Here ε_k is given by

$$\varepsilon_k = \left\{ 1 - u \frac{k(k+1)}{k_e l_1 - k_p l_1} \left(\frac{\omega_p}{\omega}\right)^2 \frac{\hat{H}_k(k_e l_1)}{\hat{H}'_k(k_e l_1)} \frac{h_k^{(2)}(k_p l_1)}{h_k^{(2)\prime}(k_p l_1)} \right\} \quad \cdots (49)$$

Now, the radiation direction impedance $Z_k(l)$ at the outer boundary $r = l$ is expressed by eq (50) which is obtained by using the relation of e m field in the region II.

$$Z_k^+(l) = \frac{E_\theta^+(l)}{H_\varphi^+(l)} = \frac{j\eta_\beta \left\{ \frac{\hat{H}_k(\beta l)}{\hat{H}_k(\beta l)} + \frac{C_k}{b_k} \frac{\hat{J}_k'(\beta l)}{\hat{J}_k(\beta l)} \right\}}{\left(1 + \frac{C_k}{b_k}\right)} \quad \cdots (50)$$

Now by using eqs. (45) to (48) at the plasma bounday surface of $r = l_1$ and also using the boundary condition about the tangential component of e m field the constant ratio C_k to b_k is given by

$$\frac{C_k}{b_k} = -\frac{\hat{J}_k'(\beta l)}{\hat{J}(\beta l)} \left\{ \frac{\frac{n_e}{n_\beta} \varepsilon_k \hat{H}_k'(k_e l_1) \hat{H}_k(\beta l_1) - \hat{H}_k'(B l_1) \hat{H}_k(k_e l}{\frac{n_e}{n_\beta} \varepsilon_k \hat{J}_k(\beta l_1) \hat{H}'(k_e l_1) - \hat{J}_k'(\beta l_1) \hat{H}_k(k_e l_1)} \right\}. \quad \cdots (51)$$

Consequently the top impedance Z_t and the inverse impedance $\hat{Z}_m$ taking into account the sheath thickness across the top of antenna is obtained from the relation described in Sec. 3.1 by using $Z_k^+(l)_n$ eq. (50) instead of $Z_k^+ c_k$ in eq. (33)

4 Radiation Resistance Affected with Boundary

Poynting theory accounting the e.m. wave and e.p. wave in the isotropic and compressible plasma is generally expressed as (Chen 1964)

$$\nabla(E \times H^*+pV^*) = -j\omega\left\{\mu_0|H|^2-\epsilon_0|E|^2+mn_0|v|^2-\frac{1}{u^2mn_0}|p^2|\right\} \quad \ldots (52)$$

Now the Poynting vector may be transformed more simply in the following way Separating the E, V of above equations into e.m. wave and e.p wave component respectively and using the fundamental equations in plasma we have

$$S = E^e \times H^* - pV^{p*} \quad \ldots (53)$$

This equation is valid at the arbitrary points in plasma so we wish to consider the Poynting vector at the boundary of the plasma In the case where the velocity at the boundary is equal to zero (rigd boundary) i.e

$$V n = 0, \quad \ldots (54)$$

the normal component of Poynting vector can be expressed as follows in terms of the tangential component of e m field at the boundary by using the eq (52) i e

$$S_n = (E_t^e + E_t^p)H_t^* \quad (55)$$

So, we have derived the impedance equation from the relation of eq (55) for the rigid boundary case and for the case where the e m.f method may be used But in the case where $V \cdot n \neq 0$ at the plasma boundary, the Poynting vector is needed to be obtained by eq (53) Exact value of total power at the boundary of model treated in previous section is out to be given by integrating the limited value of $s \cdot n$ in the vicinity of boundary over the all aboundary surface Here we discuss only the radiation resistance by obtaining the real power at the inner far distance which is practically of interest. The total power radiated in the outer direction from the wave source in the plasma is given by

$$P = \lim_{r \to \infty} \int_0^{2\pi} d\phi \int_0^{\pi} (E_\theta^e H_\phi^* + pV_r^{p*}) r^2 \sin\theta d\theta, \quad \ldots (56)$$

Now, the e m field at the $k_{pr} \to \infty$ is given as follows by the use of eq (25) i e

$$\left.\begin{aligned} E_\theta^e &= \frac{n_e}{2\pi}\frac{e^{-jk_p r}}{r}\sum_{1,3,\ldots}\frac{j^{(k+1)}}{k(k+1)}\frac{b_k}{H_k(k_e l)}\cdot\frac{d}{d\theta}P_k(\cos\theta)+\left(\frac{1}{r^2}\right) \\ H_\phi^* &= \frac{E_\theta^{e*}}{\eta_e}\cdot b_k \qquad \ldots (42) \end{aligned}\right\} \quad \ldots (57)$$

By performing the integral of eq. (56) and using the orthogonality relation given by

$$\int_0^{\pi} P_n'(\cos\theta)P_m'(\cos\theta)\sin\theta d\theta = \begin{cases} \dfrac{2n(n+1)}{2n+1}, & (n=m) \\ 0, & n \neq m \end{cases} \quad \ldots \quad (58)$$

The radiation resistance for e.m. wave is expressed by eq. (59) for the case of very thin antenna i e

$$R_e^m = \frac{n_e}{\pi} \sum_{1,3,\ldots} \frac{2k+1}{k(k+1)} \left| \frac{1}{\delta(u)\hat{H}_k(k_e l)} \right|^2 \quad (59)$$

$$R_e^m = \sin^2 \beta l R_e^l$$

Here R_e^l is the input radiation resistance of the component of e m wave

Now, we obtain the radiation resistance for e p wave The electron velocity at the inner far distance ($k_{pr} \gg 1$) is expressed by eq (60) with the help of eqs. (59) and (39) i e

$$V_r^p = \frac{c_0}{\omega n_0 m e} \frac{e^{-jk_p r}}{r} \sum_{1,3} j^{(k+1)} \frac{B_R}{h_k^{(2)}(k_p l)} P_K(\cos\theta) + 0\left(\frac{1}{r^2}\right), \quad \ldots \quad (60)$$

where B_k is constant which is given by eqs (40) and (42) while electron pressure p is obtained by using the continuity equation and state equation when electron velocity is given Now, when $k_p r >> 1$ we get

$$p = \frac{ju^2 m n_0}{\omega} \nabla V$$

$$= \frac{k_p u^2 \epsilon_0}{\omega^2 c} \frac{e^{-jk_p r}}{\gamma} \sum_{1,3} j^{(k+1)} \frac{B^k}{h_k^{(2)}(k_p l)} P_k(\cos\theta) + 0\left(\frac{1}{r^2}\right) \quad \ldots \quad (61)$$

Now, if the total power e p. wave being radiated from antenna is calculated by the product of p and $V_\gamma p^*$ ($V_\gamma p^*$ is the conjugate of V_γ^p) by using the orthogonality relation given by

$$\int_0^{\pi} P_n(\cos\theta) P_m(\cos\theta)\sin\theta d\theta$$

$$= \begin{cases} \dfrac{2}{2n+1}, & (n=m) \\ 0, & (n \neq m) \end{cases} \quad \ldots \quad (62)$$

the radiation resistance for e.p. wave may be represented by

$$R_p{}^i = \frac{\eta_e}{\pi} \cdot \frac{c}{u} \left(\frac{\omega_p}{\omega}\right)^2 \frac{|u|^2}{(k_p l)^4} \sum_{13} (2k+1) \frac{1}{[\delta_k(u)]h_k{}^{(2)\prime}(k_p l)} \quad \cdots (63)$$

$$R_p{}^m = \sin^2\beta l R_p{}^i$$

where $R_p{}^i$ is the input radiation resistance for e.p. wave.

5. Numerical Calculations

We wish to show a few examples of numerical calculation about the above analytical results Figure 4 shows radiation resistance for e m wave and e.p. wave with the parameter U that represents the hardness of the boundary. Solid line shows the characteristics of the case where the plasma boundary is spherical surface $2l$ in diameter (l = 1/2 antenna length) and dash line shows that of dipole antenna shows the characteristics of the case where the plasma boundary is spherical surface $2l$ in diameter (l = 1/2 antenna length) and dash line shows that of dipole antenna without sheath It is seen that R_{pm} without sheath is greater than that with sheath by 10 ~ 15 times. Furthermore when the boundary exists and it is a hard boundary, a few change of resistance is observed by the change of the phase of U.

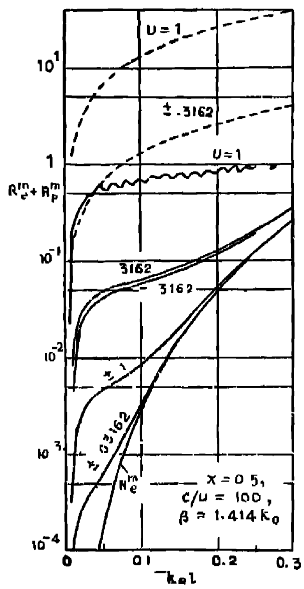

Fig. 4. Radiation resistance as a function of kel

Figure. 5 shows the change of the radiation resistance for e.p. wave when the sheath thickness is increased from the top of antenna. Under the example of numerical calculation performed here R_e^m for e m wave is not much affected by U. In the table 1 calculated values are listed

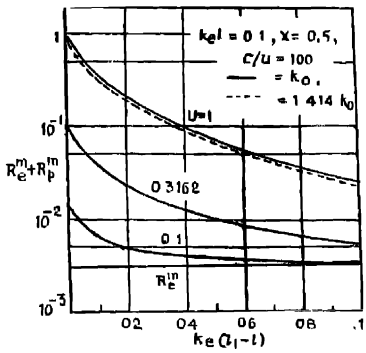

Fig 5. Relationship between radiation resistance and sheath thickness

Table 1 Calculated values of R_e^m

$\omega_p^2/\omega^2 = 0.5$, $c/u = 100$

kel	$u = 1$	$u = 0\ 1$	$u = 0.01$	$u = 0$
0 1	0.0028714	0.0028492	0 0028458	0 0028454
0 5	2.1003	2.1035	2.0291	2.0284
1 0	38 339	37 684	37 619	37 612
1.5	90.153	89 987	89.970	89.968
2 0	39 591	39.681	39 690	39.691
2 5	48 273	47 824	47 780	47.775
3.0	132.32	132.44	132 45	132.46

6. Conclusions

Here we have derived the equations by which the effect of hardness boundary and sheath thickness or dielectric surrounding the inner thin biconical antenna in a compressible plasma can be studied on the radiation characteristics for e p. wave and e.m wave From the numerical results we have seen that the change of radiation characteristics for e.m wave by the hardness U is negligibly small The change is almost a few percent only For example when $\frac{\omega_p^2}{\omega^2} = 0.5$ $\frac{c}{u} = 10^2$, that change is of the order of few percent. On the other hand the value of the radiation resistance for e p wave is changed by U especially when the boundary is soft and then the radiation resistance is proportonal to $|u|^2$ This also shows that the radiation resistance for the soft boundary does not relate to phase of U If we can obtain the value of U concretely, we are able to design the antenna which does not almost radiate the e p wave

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